2.7. Worked Examples for Gear Tooth Stresses, Dynamic Load and Wear

Problem 8

A cast steel pinion (σ_0 = 103 MNm^{-2}) rotating at 500 rpm drives a cast steel gear of a different grade (σ_0 = 140 MNm^{-2}) at a velocity ratio of 3 to 1. The pinion has 20 teeth of module 10 mm and the pressure angle is 20^0. Given that the face width of both gears is 80 mm, determine the amount of power that can be transmitted by the gears when considering the strength of the gears.

Solution

Data:

σ_{0p} = 103 MNm^{-2}
σ_{0g} = 140 MNm^{-2}
N_p = 500 rpm
i = 3
z_p = 20 teeth
m = 10 mm
$\phi = 20^0$

The first step in the analysis is to determine whether the pinion or the gear is the weaker of the pair of gears.

$$i = \frac{z_g}{z_p}$$

$$\Rightarrow 3 = \frac{z_g}{20}$$

$$\Rightarrow z_g = 3 \times 20 = 60$$

Therefore there are 60 teeth on the gear.

Comparing the pinion and the gear:

	z	σ_0 (Pa)	Y (from Table 2.1)	σ_0Y (Pa)
Pinion	20	103x10^6	0.102	10.51x10^6
Gear	60	140x10^6	0.134	18.76x10^6

As can be shown by Equation (2.13) the load capacity is proportional to σ_0Y, therefore the pinion is the weaker of the two gears and all design equations should be based on the pinion, as it has a lower σ_0Y value.

In order to determine the allowable stress for use in Equation (2.13), the pitch line velocity must first be calculated in order to determine the correct Barth equation to use (Equation (2.16)).

$$v = \omega_p \times r_p$$

$$\omega_p = \frac{2\pi N_p}{60}$$

$$= \frac{2\pi 500}{60}$$

$$= 52.36 rads^{-1}$$

$$r_p = \frac{D_p}{2}$$

$$= \frac{m \times z_p}{2}$$

$$= \frac{20 \times 10}{2} = 100mm = 0.1m$$

Therefore, $v = 52.36 \times 0.1 = 5.24ms^{-1}$

Therefore, since v < 10 ms^{-1}, the allowable stress:

$$\sigma_t = \sigma_0 \left(\frac{3}{3+v} \right)$$

$$= 103 \times 10^6 \left(\frac{3}{3+5.24} \right)$$

$$= 37.52 \times 10^6 \, Nm^{-2}$$

Therefore, according to Equation (2.15):

$$F_t = \sigma_t bYp_c$$

Where:

$$p_c = \pi.m$$

$$= \pi.10$$

$$= 31.42mm$$

$$\Rightarrow F_t = 37.52 \times 10^6 .80 \times 10^{-3} .0.102.31.42 \times 10^{-3}$$

$$= 9{,}617.96N$$

$$= 9.62kN$$

Therefore, from Equation (2.7):

$$P = F \times v$$

$$= 9617.96 \times 5.24$$

$$= 50{,}359.50W$$

$$= 50.36kW$$

Problem 9
A cast steel pinion (σ_0 = 98 MNm^{-2}) is rotating at 850 rpm and is driving a cast iron gear (σ_0 = 60 MNm^{-2}) at a speed of 136 rpm. The stub type teeth on these gears have a pressure angle of 20^0 and are required to transmit a maximum power of 23 kW. Determine the required module, number of teeth and face width for these gears considering the strength, dynamic load and wear requirements of the gears. For the consideration of wear, the elastic modulus of the steel is 200 GPa, the elastic modulus of the iron is 110 GPa and the average Brinell Hardness Number of the gear and pinion is 250 BHN.

Solution

Data:
σ_{0p} = 98 MNm^{-2}
σ_{0g} = 60 MNm^{-2}
N_p = 850 rpm
N_g = 136 rpm
$\phi = 20^0$
P = 23 kW = 23 x 10^3 W
E_p = 200 GPa
E_g = 110 GPa
BHN = 250

As the module, gear diameters and number of teeth are unknown; a starting point is to assume the number of teeth on each of the pinion and the gear. As the velocity ratio is 850/126 = 6.25, it is clear that an odd number of teeth on the pinion will not give a whole number of teeth on the gear, therefore an even number of teeth is selected from the list of teeth numbers in Table 2.1, starting with 12.

Number of Teeth on Pinion	Number of Teeth on Gear
12 x (6.25) =	75
14	87.5
16	100

As stated in Section 1.4 the minimum number of teeth is normally set at 14, but as this does not allow a whole number of teeth for the gear, the number of teeth on the pinion is selected as 16, and the number of teeth on the gear is selected as 100.

The next step in the analysis is to determine whether the pinion or the gear is the weaker of the pair of gears.
Comparing the pinion and the gear:

	z	σ_0 (Pa)	Y (from Table 2.1)	σ_0Y (Pa)
Pinion	16	98x10^6	0.115	11.27x10^6
Gear	100	60x10^6	0.161	9.66x10^6

As can be shown by Equation (2.13) the load capacity is proportional to σ_0Y, therefore the gear is the weaker of the two gears and all design equations should be based on the gear, as it has a lower σ_0Y value.

Since the gear diameters are unknown, the Equation (2.20) form of the Lewis equation is used, remembering to substitute the gear torque and number of teeth:

$$\sigma_t = \frac{2T_g}{\pi^2 kYm^3 z_g}$$

Therefore:

$$T_g = \frac{P \times 60}{2\pi \times N_g}$$

$$= \frac{23 \times 10^3 \times 60}{2\pi \times 136}$$

$$= 1614.95Nm$$

Remembering that the initial value of k is always assumed to be 4:

$$\sigma_t = \frac{2 \times 1614.95}{\pi^2 4.0.161.100.m^3}$$

$$= \frac{5.0816}{m^3}$$

As the diameters are still unknown, the pitch line velocity is not known, and therefore the first estimate for the allowable stress is:

$$\sigma_t \approx 0.5\sigma_0 = 0.5 \times 60 = 30MNm^{-2}$$

Therefore: $30 = \frac{5.0816}{m^3}$

$$\Rightarrow m = \sqrt[3]{\frac{5.0816}{30 \times 10^6}}$$

$$= 0.0055m$$

$$= 5.533mm$$

As a smaller module means a smaller and therefore cheaper gear, modules of 6 mm and possibly 5 mm will be tried first.

For m = 6 mm, D_g = 100 x 6 = 600 mm

$$v = \omega_g \times r_g$$

$$\omega_g = \frac{2\pi N_g}{60}$$

$$= \frac{2\pi 136}{60}$$

$$= 14.24rads^{-1}$$

$$r_g = \frac{D_g}{2}$$

$$= \frac{600}{2} = 300mm = 0.3m$$

Therefore, $v = 14.24 \times 0.3 = 4.27ms^{-1}$

Therefore, since v < 10 ms^{-1}, the allowable stress:

$$\sigma_t = \sigma_0\left(\frac{3}{3+v}\right)$$
$$= 60\times10^6\left(\frac{3}{3+4.27}\right)$$
$$= 24.75\times10^6\,Nm^{-2}$$

This is then compared to the induced stress:

$$\sigma_t = \frac{5.0816}{\left(6\times10^{-3}\right)^3} = 23.53\times10^6\,Nm^{-2}$$

As the induced stress is less than the allowable stress, a module of 5 mm needs to be checked to see if the smaller module is acceptable (even though it is unlikely based on the difference above), therefore:
For m = 5 mm, D_g = 100 x 5 = 500 mm

$$v = \omega_g \times r_g$$
$$\omega_g = 14.24 rads^{-1}$$ (from previous)
$$r_g = \frac{D_g}{2}$$
$$= \frac{500}{2} = 250mm = 0.25m$$

Therefore, $v = 14.24\times0.25 = 3.56ms^{-1}$

Therefore, since v < 10 ms^{-1}, the allowable stress:

$$\sigma_t = \sigma_0\left(\frac{3}{3+v}\right)$$
$$= 60\times10^6\left(\frac{3}{3+3.56}\right)$$
$$= 27.44\times10^6\,Nm^{-2}$$

This is then compared to the induced stress:

$$\sigma_t = \frac{5.0816}{\left(5\times10^{-3}\right)^3} = 40.65\times10^6\,Nm^{-2}$$

As the induced stress is much greater than the allowable stress a module of 5 mm is not acceptable.

Therefore, returning to the calculations for a module of 6 mm, the gear is stronger than necessary, so the value of k can be reduced by the ratio:

$$k = 4\times\frac{23.53}{24.75}$$
$$= 3.8028$$

Therefore:

$$b = kp_c = k\pi m$$
$$= 3.8028 \times \pi \times 6$$
$$= 71.68mm$$

Therefore, m = 6 mm, b = 71.68 mm satisfies the strength requirement.

Now the dynamic load and wear performance need to be checked so that the endurance load F_0 and the wear load F_w are calculated as the allowable values.

$$F_0 = \sigma_0 bYp_c$$
$$= \left(60 \times 10^6\right)\left(71.68 \times 10^{-3}\right)(0.161)\left(\pi 6 \times 10^{-3}\right)$$
$$= 13052.27N$$
$$= 13.05kN$$

$$F_w = D_p bKQ$$

$$D_p = m \times z_p = 6 \times 16 = 96mm$$

$$K = \frac{s_{es}^2 \sin\phi\left(1/E_p + 1/E_g\right)}{1.4}$$

$$s_{es} = 2.75(BHN) - 70$$
$$= 2.75 \times 250 - 70$$
$$= 617.5MNm^{-2}$$

$$\Rightarrow K = \frac{\left(617.5 \times 10^6\right)^2 \sin 20\left(1/\left(200 \times 10^9\right) + 1/\left(110 \times 10^9\right)\right)}{1.4}$$
$$= 1312.62kNm^{-2}$$

$$Q = 2z_g/(z_g + z_p) = 2 \times 100/(100 + 16) = 1.7241$$

$$\Rightarrow F_w = \left(96 \times 10^{-3}\right)\left(71.68 \times 10^{-3}\right)\left(1312.62 \times 10^3\right) \times 1.7241$$
$$= 15573.05N$$
$$= 15.57kN$$

Both F_0 and F_w must be greater than F_d:

$$F_d = \frac{21v\left(bC + F_t\right)}{21v + \sqrt{\left(bC + F_t\right)}} + F_t$$

Where:

$$F_t = T_g / r_g = 1614.95/0.3 = 5383.17N$$

From Figure 2.8, the permissible error for v = 4.27 ms^{-1} is 0.08 mm. For m = 6 mm, e = 0.06 mm for first class commercial gears.

From Table 2.3, steel pinion, cast iron gear, 20^0 pressure angle stub type tooth, e = 0.06 mm means that C = 486 kNm^{-1}, therefore:

$$
\begin{aligned}
F_d &= \frac{21.4.27\left(71.68\times10^{-3}\times486\times10^{3}+5383.17\right)}{21.4.27+\sqrt{71.68\times10^{-3}\times486\times10^{3}+5383.17}}+5383.17 \\
&= \frac{3606496.02}{290.22}+5383.17 \\
&= 17810.01N \\
&= 17.81kN
\end{aligned}
$$

Therefore, since $F_d > F_0$ and $F_d > F_w$, the gear is not suitable for dynamic load or for wear, therefore need to improve quality of gear manufacture and chose carefully cut gear with error of 0.03 mm for m = 6 mm.

This value required interpolation from the table, therefore:

@ 0.02 mm, C = 162 kNm^{-1}

@ 0.04 mm, C = 324 kNm^{-1}

Therefore, @ 0.03 mm; $C = \frac{0.03-0.02}{0.04-0.02}\times\left(324-162\right)+162 = 243kNm^{-1}$

Therefore:

$$
\begin{aligned}
F_d &= \frac{21.4.27\left(71.68\times10^{-3}\times243\times10^{3}+5383.17\right)}{21.4.27+\sqrt{71.68\times10^{-3}\times243\times10^{3}+5383.17}}+5383.17 \\
&= \frac{2404602.48}{240.67}+5383.17 \\
&= 13878.58N \\
&= 13.88kN
\end{aligned}
$$

Therefore, since $F_d < F_w$ and $F_d \sim F_0$ (6.05% difference) the gear is suitable for wear and within a suitable tolerance for dynamic load without requiring a precision cut gear.